Algebra A/B

Holt Public Schools Vision Statement for K-12 Mathematics Instruction:

We believe students in mathematics in Holt Public Schools need a productive disposition towards mathematics and to view themselves as confident mathematicians. In order to build this disposition, students will gain strong conceptual knowledge that then supports development of their procedural skills. Students will make sense of problems and persevere in solving them. In those problems, students will model and reason abstractly and quantitatively. Students will construct viable arguments and critique the reasoning of others.

Math

Tiered Philosophy

In Holt Public Schools, we believe all students are able to become capable mathematicians. We recognize that this does not happen at the same pace for all students, so some students, at various times, will need additional support to be successful. Because we value all students experiencing rigorous math classes with their peers, the support students receive will be in addition to their regular, at-level math course. By increasing the amount of time students engage with mathematics during the day, we are able to help students close existing knowledge gaps that hinder success with their grade level course work, see connections between mathematical ideas, deepen their understanding of current and prior knowledge, and develop a positive mathematical identity.

According to Dr. Rebecca Sarlo, Tier 2 supports and interventions at the secondary level "should be designed to support student success with core instructional content (2014)." The supports should address knowledge or gaps that are more relevant to the current core instruction students are receiving. In addition to supporting students' acquisition of mathematical concepts, students also build their efficacy at being a successful mathematics student. This happens through increasing engagement through goal setting, high quality and high frequency feedback, and students monitoring their own progress.

Students who receive this support at grades 7-9 typically have some gaps in their prior knowledge or underdevelopment of some mathematical habits of mind that will be problematic for future success. Students are identified using data points such as prior course failures, common unit test or exam scores, unit screeners, or teacher recommendation. By utilizing the mathematic support classes, students are engaged in mathematics for more minutes during the day than their peers, which helps to close knowledge gaps. The class sizes are smaller so students receive more frequent teacher feedback. Students engage in the mathematical practice standards and collaborate with their peers in order to become more confident in themselves as capable and successful mathematicians. Teachers organize learning opportunities for students to build their mathematical habits of exploring ideas, orienting/organizing, thinking in reverse, representing, justifying, generalizing, checking for reasonableness, and using mathematical language (Horn 2012). In order to provide these experiences, instruction is not of an "I do, we do, you do" type model.

According to Rollins (2014), support that is remediation of prior content that is not relevant to what the student is expected to do in their current math class only keeps that student behind. She advocates for addressing past conceptual and procedural knowledge gaps connected to the new learning expected students experience in their grade level math class. As a result, the learning opportunities teachers provide are centered on mathematical content that is prerequisite knowledge for what students need to be successful in their core class in real time. This helps students engage in the core instruction with their peers rather than falling further behind and waiting to catch up.

Below are student experiences and related teacher knowledge or actions from literature on best mathematical teaching practices. The resources used to compile this were:

- *Small Steps, Big Changes*, Confer and Ramirez (2012)
- *Principles to Actions*, National Council of Teachers of Mathematics (2014)
- *Adding It Up*, National Research Council (2001)
- *Strength in Numbers*, Horn (2012)

We believe all students need to understand the following expectations and engage in these actions at all grades:

Student experiences	Related teacher knowledge or actions
Students justify their mathematical arguments and critique those of others.	• Teachers keep the complexity of authentic learning tasks • Teachers anticipate and use students' errors and misconceptions as learning opportunities • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify • Teachers have multiple mathematical representations and strategies to help support students in making connections between their mathematical ideas and those of others
Students apply multiple strategies.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers gather evidence of knowledge during instruction and use assessment data strategically to help students refine their mathematical knowledge and support building connections between ideas.
Students write, talk about, and present their mathematical ideas.	• Teachers facilitate students making connections between mathematical ideas • Teachers anticipate common mathematical errors and misconceptions, and when students make these, use them as learning opportunities • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify
Students engage in solving mathematical problems with peers.	• Teachers keep the complexity of authentic learning tasks • Teachers build interdependence among students by facilitating group work and having norms.
Students engage in productive struggle and persevere.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) in order to facilitate a productive struggle • Teachers keep the complexity of authentic learning tasks to promote productive struggle • Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to know entry points into the problems and suggestions of prior knowledge that

	will help students progress through complex tasks.
Students solve complex problems with multiple solution paths.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) to allow multiple solution paths • Teachers have multiple mathematical representations and strategies to help teach students • Teachers keep the complexity of authentic learning tasks so there are multiple solution paths • Teachers gather evidence of knowledge during instruction and use assessment data strategically in order to facilitate students seeing a robust set of solution paths
Students create and use visual models and multiple representations.	• Teachers have a strong understanding of the mathematics they teach and how it connects (concepts, procedures, representations, strategies, language) to allow multiple representations • Teachers keep the complexity of authentic learning tasks
Students are self-assessing based on learning goals. Related to students use metacognitive strategies to know when to adjust their learning strategies in relation to learning goals.	• Teachers anticipate common mathematical errors and misconceptions, and when students make these, use them as learning opportunities • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges
Students value mathematics.	• Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify to provide multiple opportunities for students to see value in multiple aspects of mathematics • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges
Students believe in their own efficacy.	• Teachers facilitate a high level of student discourse, probe student thinking through purposeful questions, and ask students to justify to provide multiple opportunities for students to grow their efficacy • Teachers gather evidence of knowledge during instruction and use assessment data strategically in order to provide support to students • Teachers differentiate, when appropriate, for students who are struggling as well as those who need additional challenges • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to support all students at being successful in mathematics
Students will make connections based on **conceptual** understandings.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers facilitate students making connections between mathematical ideas • Teachers have multiple mathematical representations and strategies to help teach students • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning

Students make connections between multiple representations.	• Teachers have a strong understanding of the mathematics they teach and how it connects: concepts, procedures, representations, strategies, language • Teachers have multiple mathematical representations and strategies to help teach students • Teachers facilitate students making connections between mathematical ideas in order to connect conceptual understandings to procedural knowledge and connections across mathematical ideas • Teachers anticipate prior knowledge and common possible ways students will attempt a problem while planning in order to identify the connections students should see

Algebra A/B course overview

Algebra A/B is typically a tenth grade course, although there is flexibility regarding when the student takes the course. Algebra A/B explores multiple function families (linear, quadratic, higher order polynomials, and rational functions). Students look at the characteristics of these functions, including patterns in the tables, graph characteristics, and forms of equations, to apply these to writing and solving equations in mathematical and real-world problems.

Approximate learning timeline

Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun

Linear Functions and Bivariate Data	Quadratic Functions	Polynomial Functions	Solving f(x) = g(x) (Breaking even)	Rational Functions
Fit linear models to data that suggests a linear relationship and justify which lines make better fits; use the lines to make predictions about situations; write inverses for linear equations as a way of solving	Properties of quadratic functions' tables, graphs, and equations; apply quadratic functions to real-world situations; solve quadratic functions; work with the x-intercept, standard (simplified), and vertex form of the equation	Properties of polynomial functions' tables, graphs, and equations; apply polynomial functions to real-world situations; solve polynomial functions when given sufficient criteria/forms; explore effects of different exponents (negative and rational) on values	Solving to find the coordinate point where two functions intersect (functions limited to polynomials); apply to real-world situations	Properties of rational functions' tables, graphs, and equations; apply rational functions to real-world situations; solve rational functions;

Algebra AB

Unit: Linear

Proficiency level	Standard: **S.ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **S.ID.9** Distinguish between correlation and causation.	HPS assessment question	SAT question along with strand aligned to
Advanced	In situations, generating alternative hypothesis for associations besides causation.	According to a paper published by the New England Journal of Medicine, there is a strong correlation between the *number of Nobel Prize winners* from a particular country (measured by the number of winner per ten million people in the population) and the *amount of chocolate eaten* in that county (measured by the kg per year consumed per person). Based on this legitimate and reliable data, should we start arguing that all students should eat more chocolate? Why or why not?	
Proficient	Explain whether data has a strong or weak or no association and whether it has a positive or negative association. Explain why a level of correlation of data does not imply causation.	In the space below, sketch one scatterplot with at least 10 points (no need to identify the variables) that has an overall downward trend, and another one that has an upward trend. Make the downward-trending scatterplot <u>more</u> correlated than the other. Explain what you did to make this the case.	

Developing	Explain whether data has a strong or weak or no association OR whether it has positive or negative association.		
Beginning	Create a scatterplot for two quantitative variables.		

Proficiency level	Standard: **S.ID.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. **S.ID.6c** Fit a linear function for a scatter plot that suggests a linear association. **S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Create a reasonable	**In an experiment, a glass bottle was filled by adding 50 milliliters at a time and measuring the height of the waterline in the bottle after each**	Heart of Algebra

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<td>line of fit for data that suggests a linear association. Write an equation for the line. Relate the coefficients of the line of fit to the situation. Justify why the line is reasonable. Using the line of fit or the equation for the line of fit, solve problems within the context of the data.</td>
<td>

50 ml was added. The data table below represents this data. A student made the scatterplot and drew two lines to approximate the data.

Bottle Lab

Volume of Water (mL)	Height of Water (mm)
0	20
50	38
100	51
150	77
200	85
250	147
300	245

a. The line g(x) passes through (8, 0.69) and (50, 28.5). Find a rule for this line. Show your work.
b. What does the rate and y-intercept for the line tell you about the situation?
c. Use your line to make a prediction for the volume of water it would take to reach a height of 500 mm.

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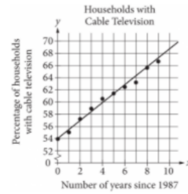
12 A cable company recorded the percentage of households in the United States that had cable television from 1987 to 1997. In the scatterplot below, *x* represents the number of years since 1987 and *y* represents the percentage of households with cable television. The line of best fit for the data is shown.



Which of the following is closest to the equation of the line of best fit shown?

A. $y = 54x + \frac{7}{5}$
B. $y = \frac{7}{5}x - 54$
C. $y = \frac{7}{5}x + 54$
D. $y = \frac{7}{5}x$

Which of the following is the best interpretation of the slope of the line of best fit shown for these data?

A. The actual increase in the percentage of households with cable television each year
B. The predicted increase in the percentage of households with cable television each year
C. The actual increase in the number of households with cable television each year
D. The predicted increase in the number of households with cable television each year

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<td>Developing</td>
<td>Create a line of fit for data that suggests a linear association. Using the line of fit or the equation</td>
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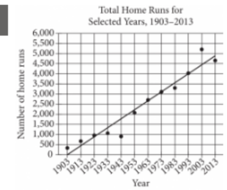
<tr><td></td><td>for the line of fit, solve problems within the context of the data.</td><td></td><td></td></tr>
<tr><td>Beginning</td><td>Given a line of fit on a scatterplot, use the line to solve problems within the context of the data.</td><td></td><td>Problem Solving and Data
9


The scatterplot above shows the total number of home runs hit in major league baseball, in ten-year intervals, for selected years. The line of best fit for the data is also shown. Which of the following is closest to the difference between the actual number of home runs and the number predicted by the line of best fit in 2003?
A. 250
B. 500
C. 750
D. 850</td></tr>
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Proficiency level	Standard: **A.REI.3** Solve linear equations and ~~inequalities~~ in one variable, including equations with coefficients represented by letters. **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).* **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and ~~exponential~~ functions.* (Exponential is in Algebra C.)	HPS assessment question	SAT question along with strand aligned to
Advanced	Can interpret contextual problem to write equation needed to be solved from the situation.	Neil borrowed a sum of money from his parents as an interest free loan to help him start a small business. You know that 3 months into the loan, he owed $6000 and that 27 months into the loan, he owed $4800.	

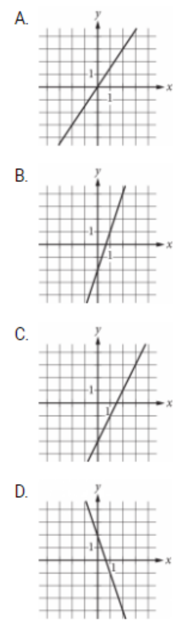
		Give Neil a way to determine the amount he still owes his parents after any number of months. How much money did Neil initially owe his parents? What amount of money is Neil paying back to his parents each month? After how many months will he have paid all the money back?	
Proficient	Find a solution to a linear equation in any form, including equations with coefficients represented by letters.	**Solve the following linear equations. Show all your work. Make sure your solutions are exact solutions.** 7(x + 13) = 22 14(x – 6) + 3 = 32 3 = 4x – 3 -25 = -5x – 4	Heart of Algebra 32 $2(5x-20)-(15+8x)=7$ What value of *x* satisfies the equation above?
Developing	Writes an inverse.	Write the rule for the inverse of the function f(x) = 5.1x – 8 Write the rule for the inverse of the function g(x) = -9(x + 1)	Passport to Advanced Math

			6 $0.8p = t$ At a store, a coat originally priced at p dollars is on sale for t dollars, and the relationship between p and t is given in the equation above. What is p in terms of t ? A. $p = t - 0.8$ B. $p = 0.8t$ C. $p = \frac{0.8}{t}$ D. $p = \frac{t}{0.8}$
Beginning	Find an approximate solution in a table or graph for a linear equation in any form.		

Proficiency level	Standard: **A.REI.3** Solve linear ~~equations~~ and inequalities in one variable, including equations with coefficients represented by letters. **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).* **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and ~~exponential~~ functions.* (Exponential is in Algebra C.)	HPS assessment question	SAT question along with strand aligned to
Advanced	Can interpret contextual problem to write inequality needed to be solved from the situation.	Tony decided to start saving money to buy a mountain bike. He started to save a portion of his paycheck in a bank account each week. After five weeks, he had $130. After eight weeks, he had $205. Assuming this situation is linear, when would he have more than $500?	
Proficient	Find a solution to a linear inequality in any form, including equations with coefficients represented by letters.		
Developing	Finds a solution to the equation not accounting for full range of solutions as related to domain and range.		

Beginning	Find an approximate solution in a table or graph for a linear equation in any form.		

Proficiency level	Standard: **CC.9-12.F.IF.7a** Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima. **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	HPS assessment question	SAT question along with strand aligned to
Advanced	Based on given rule, justify all of the characteristics below OR can describe all the possibilities for a general rule.	Neil borrowed a sum of money from his parents as an interest free loan to help him start a small business. You know that 3 months into the loan, he owed $6000 and that 27 months into the loan, he	Heart of Algebra **13** The function h, defined by $h(t) = at + b$, where a and b are constants, models the height, in centimeters, of the sunflower after t days of growth during a time period in which the growth is approximately linear. What does a represent? A. The predicted number of centimeters the sunflower grows each day during the period B. The predicted height, in centimeters, of the sunflower at the beginning of the period C. The predicted height, in centimeters, of the sunflower at the end of the period D. The predicted total increase in the height of the sunflower, in centimeters, during the period

		owed $4800. Write the rule, explain how that gives you information about the graph, and use it to find the x-intercept and y-intercept. Justify what these tell you about the situation.	
Proficient	Based on given rule, explain all of the following: x-intercept and y-intercept and find values of the coordinates. Graph (sketch or on axis) the rule.		Heart of Algebra **2** Which of the following is the graph of the equation $y = 3x - 2$ in the xy-plane? 
Developing			
Beginning			

I wasn't sure what to call a 1 or 2 here, especially with what I know they do 7-8. I also wasn't sure if the same delineation of "explain" and "justify" worked here.

Proficiency level	Standard: **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.* **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear ~~and quadratic~~ functions, ~~and simple rational and exponential functions.*~~ **CC.9-12.F.LE.1b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.* **CC.9-12.F.LE.2** Construct linear ~~and exponential~~ functions~~, including arithmetic and geometric sequences~~, given a graph, a description of a relationships, or two input-output pairs (include reading these from a table).	HPS assessment question	SAT question along with strand aligned to
Advanced	Write a linear rule in point-slope form. Isn't y-inter form more advanced?	Write the linear rule that goes through the points (12, -17) and (21.5, -10)	
Proficient	Write a linear rule from any table, graph, situation, or two pairs of points.	Write the linear rule that goes through the points	Heart of Algebra

		(12, -17) and (21.5, -10)	**5** In the xy-plane, line ℓ passes through the points $(0, 1)$ and $(1, 4)$. Which of the following is an equation of line ℓ ? A. $y = \frac{1}{3}x + 1$ B. $y = \frac{1}{3}x - 1$ C. $y = 3x + 1$ D. $y = 3x - 1$ **14** The boiling point of water at sea level is 212 degrees Fahrenheit (°F). For every increase of 1,000 feet above sea level, the boiling point of water drops approximately 1.84°F. Which of the following equations gives the approximate boiling point B, in °F, at h feet above sea level? A. $B = 212 - 1.84h$ B. $B = 212 - (0.00184)h$ C. $B = 212h$ D. $B = 1.84(212) - 1{,}000h$
Developing	Write a linear rule from any 2 or 3 of the above representations.		
Beginning	Write a linear rule from any 1 of the above representations.		

Unit: Quadratics

Proficiency level	Standard: **A.REI.4** Solve quadratic equations in one variable. **CC.9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring **(honors only)**, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b* **(honors only).** **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment question	SAT question along with strand aligned to
Advanced	Find all real solutions to any quadratic equation with generalized coefficients in any form or when no solution exists. (And all of the 3 level.)		
Proficient	Find all real solutions to a quadratic equation in any form or when no solution exists. Relate the solutions or lack of solutions to characteristics of the function or inverse.	Solve the following. a. $31 = 7(x - 4)^2 + 9$ b. $31 = 7x^2 - 4x + 9$	Passport to Advanced Math

	For honors: Find all complex solutions to a quadratic equation in any form.	Explain how to determine if a quadratic equation has 0, 1, or 2 solutions.	13 $2x^2 - 4x = t$ In the equation above, t is a constant. If the equation has no real solutions, which of the following could be the value of t? A. −3 B. −1 C. 1 D. 3 7 The functions f and g are defined by $f(x) = 4x$ and $g(x) = x^2$. For what value of x does $f(x) - g(x) = 4$? A. −2 B. −1 C. 1 D. 2
Developing	Find all real solutions to a quadratic equation in one form OR find one solution to a quadratic equation in any form.		Passport to Advanced Math 4 $x(x+2) = 8$ Which of the following lists all solutions to the quadratic equation above? A. 8 and 6 B. 4 and −2 C. −4 and 2 D. $\sqrt{6}$
Beginning	Find an approximate solution in a table or graph for a quadratic equation in any form.		

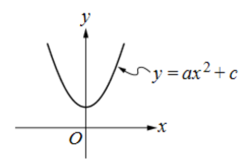
*HHS math dept chooses to use inverses as the strategy to solve because it transfers to multiple function families. However, if a student factors or uses the quadratic formula as a solving strategy, they are still capable of reaching proficiency.

Proficiency level	Standard: **CC.9-12.A.REI.4a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)^2 = q that has the same solutions. Derive the quadratic formula from this form **(honors only, if time allows)**. **F.IF.8** Write a function defined by an expression in different but equivalent forms ~~to reveal and explain different properties of the function.~~ (The part in the strike through will show up in another learning progression.) **CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A.SSE.2** Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Given a quadratic rule in any form, transform it into vertex form.	Put the following rules in vertex form $f(x) = 2x^2 - 17x + 3$ $g(x) = -4x^2 + 15x$ $h(x) = 7(x - 4)(x + 9)$	Not assessed directly, but students would need to do this to solve a quadratic, standard A.REI.4, or find the vertex, standard A.SSE.3b.
Developing	Can convert from x-intercept or standard form to vertex form (but not both).		

Beginning	Demonstrate conceptual understanding of process by setting up, but can't carry out the procedure.		

Proficiency level	Standard: **CC.9-12.A.SSE.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **F.IF.8** Write a function defined by an expression in different but equivalent forms ~~to reveal and explain different properties of the function.~~ (The part in the strike through will show up in another learning progression.) **CC.9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **A.SSE.2** Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Given a quadratic rule in any form, find the vertex for the function symbolically.	Give the coordinates for the vertex for the following:	Passport to Advanced Math

		$f(x) = 2x^2 - 17x + 3$ $f(x) = -8(x + 1)^2 - 2$ $f(x) = -4x^2 + 15x$	**26** The gas mileage $M(s)$, in miles per gallon, of a car traveling s miles per hour is modeled by the function below, where $20 \le s \le 75$. $M(s) = -\frac{1}{24}s^2 + 4s - 50$ According to the model, at what speed, in miles per hour, does the car obtain its greatest gas mileage? A. 46 B. 48 C. 50 D. 75
Developing	Can find the x-coordinate of the vertex.		
Beginning	Demonstrate conceptual understanding of process by setting up, but can't carry out the procedure.		

Proficiency level	Standard: **CC.9-12.F.IF.7a** Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima. **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	HPS assessment question	SAT question along with strand aligned to
Advanced	Based on given rule, justify all of the characteristics below OR can describe all the possibilities for a general rule.		Passport to Advanced Math 11  The vertex of the parabola in the *xy*-plane above is $(0, c)$. Which of the following is true about the parabola with the equation $y = -a(x-b)^2 + c$? A. The vertex is (b, c) and the graph opens upward. B. The vertex is (b, c) and the graph opens downward. C. The vertex is $(-b, c)$ and the graph opens upward. D. The vertex is $(-b, c)$ and the graph opens downward.

Proficient	Based on given rule, explain all of the following: x-intercept(s), y-intercept, and vertex and find values of the coordinates. Graph (sketch or on axis) the rule.	**Write a rule for the following conditions.** An parabola that opens down with a vertex at (7, -2) A parabola with a rate that changes at a constant rate of –14 and y-intercept of (0,24) A quadratic with a vertex at (-9, 2) and opens down. A quadratic with a vertex at (-10, -4.5) that opens up. An increasing linear equation with y-intercept at (0, 7). A decreasing linear equation with a y-intercept at (0, -6) **Find the vertex, x-intercept, and y-intercept for $4x^2 - 6x - 8$**	Passport to Advanced Math **22** The graphs in the *xy*-plane of the following quadratic equations each have *x*-intercepts of -2 and 4. The graph of which equation has its vertex farthest from the *x*-axis? A. $y = -7(x+2)(x-4)$ B. $y = \frac{1}{10}(x+2)(x-4)$ C. $y = -\frac{1}{2}(x+2)(x-4)$ D. $y = 5(x+2)(x-4)$
Developing	Based on given rule, determine most of the following: x-intercept(s), y-intercept, and vertex and find values of the coordinates. Graph (sketch or on axis) the rule.		
Beginning	Graph a quadratic function given as a table of values, including the intercepts and vertex on the graph. OR		

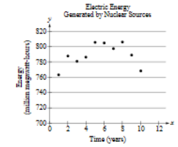
	Based on given rule, determine any of the following: x-intercept(s), y-intercept, and vertex and find values of the coordinates. Graph (sketch or on axis) the rule.		

Explain: "It's the vertex because it's the max/min," is a 3 because it gets at why the kid gave that answer (odd or even degree).
Justify: "It's the vertex because the y-values are symmetric around this point. The distance away (as found by the difference before squaring) is the same for each of the x-values on either side," is a 4 because it delves further into the mathematical reason.

CC.9-12.F.IF.8a

Use the process of factoring **(honors only)** and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. I feel like this standard could be included with the graphing (since kids would need to do this to find the values they mark) or the converting forms or solving – all depending on the question asked of kids. Perhaps writing a separate one for this is helpful too though for separating out SBG.

Proficiency level	Standard: **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.* **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions, ~~and simple rational and exponential functions.*~~	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Write a quadratic rule from any table, graph, or situation.	A Food Network Challenge show had a pizza throwing contest. The contestants had to toss pizza dough into the air. The highest the pizza dough was tossed was 21 feet high. It took 0.95 seconds to reach that height. Recall for gravity situations the *a*-value is -16. Write a rule where the independent variable is time and the dependent variable is the height of the pizza dough.	Passport to Advanced Math **30** A poster has an area of 432 square inches. The length x, in inches, of the poster is 6 inches longer than the width of the poster. Which of the following equations can be solved to determine the length, in inches, of the poster? A. $x^2 - 6 = 432$ B. $x^2 - 6x = 432$ C. $x^2 + 6 = 432$ D. $x^2 + 6x = 432$ **22** The graphs in the xy-plane of the following quadratic equations each have x-intercepts of -2 and 4. The graph of which equation has its vertex farthest from the x-axis? A. $y = -7(x+2)(x-4)$ B. $y = \frac{1}{10}(x+2)(x-4)$ C. $y = -\frac{1}{2}(x+2)(x-4)$ D. $y = 5(x+2)(x-4)$ Problem Solving and Data

			30 The scatterplot below shows the amount of electric energy generated, in millions of megawatt-hours, by nuclear sources over a 10-year period.  Of the following equations, which best models the data in the scatterplot? A. $y = 1.674x^2 + 19.76x - 745.73$ B. $y = -1.674x^2 - 19.76x - 745.73$ C. $y = 1.674x^2 + 19.76x + 745.73$ D. $y = -1.674x^2 + 19.76x + 745.73$
Developing	Can write a quadratic rule in one form.		
Beginning	Write a quadratic rule when all information given (ex. The vertex and the constant second change.)		

I at first made a 2 where they were given some of the info and had to find the rest (ex: given the x-intercepts but had to find constant second change), but realized that would probably be the case for any proficient spot. Then I wondered about only being able to do it from one rep (table, graph, or situation) or only being able to write in one form. For the 4, I wondered if we ever gave them tables where delta x was NOT 1.

Unit: Polynomials

I feel like we do more than this in the unit, but the other standards are "know" or "understand." I figured we'd need one for "write" and "solve," so I started those. I also wondered about a progression to address the distributive property and like terms (equivalent forms, perhaps).

Proficiency level	Standard: **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **F.IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	HPS assessment question	SAT question along with strand aligned to
Advanced	Based on given rule, justify all of the characteristics below OR can describe all the possibilities for a general rule.		

<table>
<tr><td>Proficient</td><td>Based on given rule, explain all of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, possible number of relative extrema, and the number of differences in the table pattern before a constant change is found. Graph (sketch or on axis) the rule.</td><td rowspan="3">What is/are the x-intercept(s) of the function $f(x)=15(x-3)(x-4)(x+18)(x+12)$?

If a polynomial has 2 local minima, 1 local maxima, and 4 x-intercepts, what is the sign on the leading coefficient (positive or negative) and what is the degree (even or odd)?

What is the table pattern for the polynomial $k(x)=-3x^{13}+9x^{2}+11x+9$?

Which polynomial does <u>not</u> have a left end that points up and a right end that points down?

What degree is the polynomial $12+3x-4x^{2}+10x^{3}+x^{4}-2x^{5}$?

What degree is the polynomial -6(x + 5)(x – 2)(x + 9)(x - 9)?

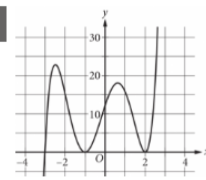
A polynomial with a positive a value (leading coefficient) <u>must</u> have what characteristic?

How many y-intercepts does -3(x + 2)(x - 5)(x + 11) have?

How many x-intercepts does -3(x + 2)(x - 5)(x + 11) have?

How many total local minimums and maximums does -3(x + 2)(x - 5)(x + 11) have?</td><td></td></tr>
<tr><td>Developing</td><td>Based on given rule, determine most of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, possible number of relative extrema, and the number of differences in the table pattern before a constant change is found. Graph (sketch or on axis) the rule.</td><td></td></tr>
<tr><td>Beginning</td><td>Based on given rule, determine any of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, possible number of relative extrema, and the number of differences in the table pattern before a constant change is found. Graph (sketch or on axis) the rule.</td><td></td></tr>
</table>

Explain: "Because it's odd degree, the end behavior is opposite," is a 3 because it gets at why the kid gave that answer (odd or even degree).
Justify: "If the inputs become really big positive numbers, the outputs get increasingly larger. When the inputs become really large negative numbers, the outputs get increasingly smaller. As a result, the end behavior is in opposite directions," is a 4 because it delves further into the mathematical reason.

Proficiency level	Standard: **CC.9-12.F.BF.1** Write a function that describes a relationship between two quantities.* **CC.9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear and~~ quadratic functions, ~~and simple rational and exponential functions.*~~	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Write a polynomial rule from a table, graph, or situation with sufficient information given or able to be calculated.	Write rules that fit the following specifications. Explain why your rule works. If it is not possible, explain why. a. A 3rd degree polynomial with no x-intercepts. b. A rule that would give the graph shape shown to the right. c. A 4th degree polynomial with x-intercepts **only** at (9,0), (-4, 0), and (1.5, 0). d. A second degree polynomial with one x-intercept at (100, 0). e. A fourth degree polynomial with x-intercepts only at (0, 0) and (-6, 0). f. A third degree polynomial that increases as x-values get really large and has three x-intercepts.	Passport to Advanced Math 12  The graph of the function *f* is shown in the *xy*-plane above, where $y = f(x)$. Which of the following functions could define *f* ? A. $f(x) = (x-3)(x-1)^2(x+2)^2$ B. $f(x) = (x-3)^2(x-1)(x+2)$ C. $f(x) = (x+3)(x+1)^2(x-2)^2$ D. $f(x) = (x+3)^2(x+1)(x-2)$
Developing	Write a polynomial rule from two of the representations (table, graph, or situation) with sufficient information given or able to be		

	calculated.		
Beginning	Write a polynomial rule from one of the representations (table, graph, or situation) with sufficient information given or able to be calculated.		

Proficiency level	Standard: **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment question	SAT question along with strand aligned to
Advanced			
Proficient	Find all real solutions to a polynomial equation in any form or when no solution exists. Relate the solutions or lack of solutions to characteristics of the function or inverse. For honors: Find all complex solutions to a quadratic equation in any form.	Use your inverse to solve the following functions without having to search. If there are no solutions, explain why. 1. $-12 = -4x^5 - 19$ 2. $0 = 2x^{10} - 4$ 3. $0 = 4x^3 + 6x - 19$ 4. $-12 = -4x^6 - 19$ 5. $0 = (x - 5)(7x + 14)(-9x + 22)$ 6. $10 = 5x^4 + 2x^2$	Passport to Advanced Math 24 The polynomial $p^4 + 4p^3 + 3p^2 - 4p - 4$ can be written as $(p^2 - 1)(p + 2)^2$. What are all of the roots of the polynomial? A. −2 and 1 B. −2, 1, and 4 C. −2, −1, and 1 D. −1, 1, and 2

Developing	Write the inverse rule for polynomials when possible.	Which expression gives the inverse of the function $f(x) = 2x^5 + 1$?	
Beginning	Know that finding the x-intercepts of a factored polynomial equation is the same as finding the solutions when f(x) = 0. Know the conditions for which a polynomial equation can have an inverse rule written.	What is/are the x-intercept(s) of the function $f(x) = 15(x-3)(x-4)(x+18)(x+12)$?	

It's not really "any" form for these…and does honors still do complex here too?

Unit: f(x)=g(x)

Proficiency level	Standard:**A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* **A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), ~~focusing on pairs of linear equations in two variables.~~ **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and ~~exponential~~ functions.* (Exponential is in Algebra C.) **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) =*	HPS assessment question	SAT question along with strand aligned to

	(x+1)/(x-1) for x ≠ 1 (x not equal to 1).		
Advanced	Solving complex, multi-step problems. (Example vertex form=vertex form.)	A teacher is going into the kayak building business. To start, she would have to buy a saber-saw for $215, wood clamps for $100, a belt sander for $150, and a ventilator (to keep from breathing epoxy fumes) for $75. The materials for each boat will cost about $635, so she plans to sell the boats for $700 each. Find the minimum number of boats the teacher would have to make and sell so that her business will become profitable. You must use at least two representations (table, graph, rule) to justify your answer. Solve $8(x-4)^2 - 10 = -2(x+5)^2 + 40$	
Proficient	Find all real solutions or when no solution exists to a f(x)=g(x) where f(x) and g(x) are a polynomial in any form. Relate the solutions or lack of solutions to characteristics of the function or inverse, or the inability to symbolically find solutions to characteristics of the rule. (In rational function unit, same standard but f(x) and g(x) can be rational functions.)	Solve: $4(x+2)(-3x+5) = 2x - 7$ $-15.8x^5 + 3x - 2 = 3x + 9$ $-15.8x + 3 = 6x - 12$	Heart of Algebra **6** If $x+3=2x-2$, what is the value of $x-4$? A. 9 B. 5 C. 4 D. 1 Passport to Advanced Math

			25 $y = 2x + 4$ $y = (x - 3)(x + 2)$ The system of equations above is graphed in the *xy*-plane. At which of the following points do the graphs of the equations intersect? A. $(-3, -2)$ B. $(-3, 2)$ C. $(5, -2)$ D. $(5, 14)$ **17** $(x - 1)^2 = 3x - 5$ What is one possible solution to the equation above?
Developing	Find some but not all real solutions or incorrect solutions but the work shows conceptual and procedural understanding.		
Beginning	Find an approximate solution in a table or graph for a f(x)=g(x) where f(x)--g(x) is invertible.		

Unit: Rationals

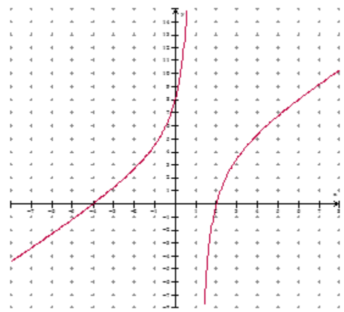
Proficiency level	Standard: **A.APR.6** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. **F.IF.8** Write a function defined by an expression in different but equivalent forms ~~to reveal and explain different properties of the function.~~ (The part in the strike through will show up in another learning progression.)	HPS assessment question	SAT question along with strand aligned to
Advanced	Do both of the directions with generalized coefficients.		
Proficient	Given a rational function in a(x)/b(x), rewrite as q(x) + r(x)/b(x). Given q(x) + r(x)/b(x), rewrite as a(x)/b(x).	Rewrite the function $b(x)=\dfrac{2x^2+15x-3}{x+5}$ in the form $f(x)=g(x)+\dfrac{r(x)}{q(x)}$. Rewrite the function $f(x)=13x^2-6+\dfrac{11}{3x+2}$ in the form $f(x)=\dfrac{g(x)}{h(x)}$. If $x\neq 0.5$, $\dfrac{6x^2+9x-6}{2x-1}$ is equivalent to:	Passport to Advanced Math **12** Which of the following is equivalent to $\dfrac{4x^2+6x}{4x+2}$? A. x B. $x+4$ C. $x-\dfrac{2}{4x+2}$ D. $x+1-\dfrac{2}{4x+2}$
Developing	Do one of the directions from 3 above.		

		Find the remainder when $8x^3 + 4x^2 - 6x + 2$ is divided by $2x^2$.	
Beginning	Demonstrate conceptual understanding of process by setting up, but can't carry out the procedure.		

Proficiency level	Standard: **A.REI.2** Solve simple rational and ~~radical equations~~ in one variable, and give examples showing how extraneous solutions may arise. **CC.9-12.A.APR.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **F.BF.4a** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).*	HPS assessment question	SAT question along with strand aligned to
Advanced	Can solve rational equations composed of any polynomial expressions.	Solve: $\frac{4}{6x-7} = \frac{5x+1}{-3x+11}$	
Proficient	Solve a rational equation in any form. Relate the solutions or lack of solutions to characteristics of the original functions in the equation.	Solving questions $5+\frac{2}{x-4} = -2$ $2x+\frac{4}{x+3}=0$	Heart of Algebra 13 $g(t)=\frac{5(7t-12c)}{2}-25$ The number of people who go to a public swimming pool can be modeled by the function *g* above, where *c* is a constant and *t* is the air temperature in degrees Fahrenheit (°F) for $70 < t < 100$. If 350 people are predicted to go to the pool when the temperature is 90 °F, what is the value of *c* ? A. 20 B. 40 C. 60 D. 80

			31 A group of friends decided to divide the $800 cost of a trip equally among themselves. When two of the friends decided not to go on the trip, those remaining still divided the $800 cost equally, but each friend's share of the cost increased by $20. How many friends were in the group originally?
Developing	Find one solution to a rational equation in any form when there are more than one OR find all solutions when one is not in the domain OR can only find when given the equation in f(x)/g(x) form		
Beginning	Find an approximate solution in a table or graph for a rational equation in any form.		

Proficiency level	Standard: **F.IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.** **CC.9-12.F.IF.7d** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **CC.9-12.F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	HPS assessment question	SAT question along with strand aligned to
Advanced	Based on given rule, justify all of the characteristics below OR can describe all the possibilities for a general rule.		
Proficient	Based on given rule, explain all of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, and vertical and horizontal asymptotes and their values. Graph (sketch or on axis) the rule.	What feature of graph for $b(x)=\frac{x^2+5x-12}{4x^3+3}$ can you find by solving the equation $4x^3+3=0$? Which of the following rational functions has end behavior described by a horizontal asymptote? Identify all of the discontinuities for the	
Developing	Based on given rule, determine most of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, and vertical and horizontal asymptotes		

	and their values. Graph (sketch or on axis) the rule.	graph of $a(x)=\frac{6x-1}{(x-1)(x+2)}$.	
Beginning	Based on given rule, determine any of the following: possible number of x-intercepts and find coordinates of them when possible, the y-intercept, end behavior, and vertical and horizontal asymptotes and their values. Graph (sketch or on axis) the rule.	What are the x-intercepts of $a(x)=\frac{6x-1}{(x-1)(x+2)}$ The graph below is made by a rational function. It has: • x-intercepts at (-4,0) and (2,0) • y-intercept at (0,8) • a vertical asymptote at x = 1 Which of the following statements could be true of the rule $y=\frac{f(x)}{g(x)}$?  Which function below will have a graph that has a discontinuity at $x=-4$, a horizontal asymptote at $y=1$, and a y-intercept at (0, 2)? Which of the following is **not** true about the function f(x) = $\frac{2}{(x+5)}-4$?	

Explain: "Because it's odd degree, the end behavior is opposite," is a 3 because it gets at why the kid gave that answer (odd or even degree). Justify: "If the inputs become really big positive numbers, the outputs get increasingly larger. When the inputs become really large negative numbers, the outputs get increasingly smaller. As a result, the end behavior is in opposite directions," is a 4 because it delves further into the mathematical reason. Do you want these updated to be rational specific?

Proficiency level	Standard: **CC.9-12.N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* **CC.9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	HPS assessment question	SAT question along with strand aligned to
Advanced	Rewrite and simplify expressions with rational and negative exponents using the properties of exponents.		
Proficient	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	Rewrite $f(x) = \frac{1}{x^9}$ in the form $f(x) = x^n$: Rewrite $f(x) = \sqrt[7]{x^8}$ in the form $f(x) = x^n$ Rewrite $f(x) = \frac{1}{\sqrt[7]{x^8}}$ in the form $f(x) = x^n$ Rewrite $f(x) = x^{-11}$ in an equivalent form that has no	Passport to Advanced Math **20** If $2\sqrt{2x} = a$, what is $2x$ in terms of a? A. $\frac{a}{2}$ B. $\frac{a^2}{4}$ C. $\frac{a^2}{2}$ D. $4a^2$

		fractional or negative exponents. Rewrite $f(x) = x^{\frac{1}{10}}$ in an equivalent form that has no fractional or negative exponents. Rewrite $f(x) = x^{-\frac{7}{4}}$ in an equivalent form that has no fractional or negative exponents.	**16** If $a^{\frac{b}{4}} = 16$ for positive integers *a* and *b*, what is one possible value of *b* ?
Developing			
Beginning			

Parent Resources:

A prototypical problem that is quintessential to the main idea of each unit:

Linear Functions

Write a linear rule for the function that goes between (52, 200) and (80, 120)

Linearish Data

Find the total squared error for y = 6x + 11 compared to the data (-5, -15), (0, 10), (8, 55), (11, 80), and (20, 135)

Quadratic Functions

Put $8x^2 + 20x - 30$ in vertex form.

Inverses of Quadratics

Solve $8x^2 + 20x - 30 = 100$.

Polynomial Functions

List the properties (table pattern, graph shape including the most number of max/mins and correct end behavior, possible number of y-intercepts and x-intercepts) of $(2x^2 + 3)(-5x^3 - 2x + 1)$.

Breaking Even

Solve $8x^2 + 20x - 30 = -6x + 10$

Rational Functions

Give the vertical asymptote, end behavior, x-intercepts, y-intercept for $(9x^2 + 10)/(x - 7)$
And then solve for what x gives an answer of 50.